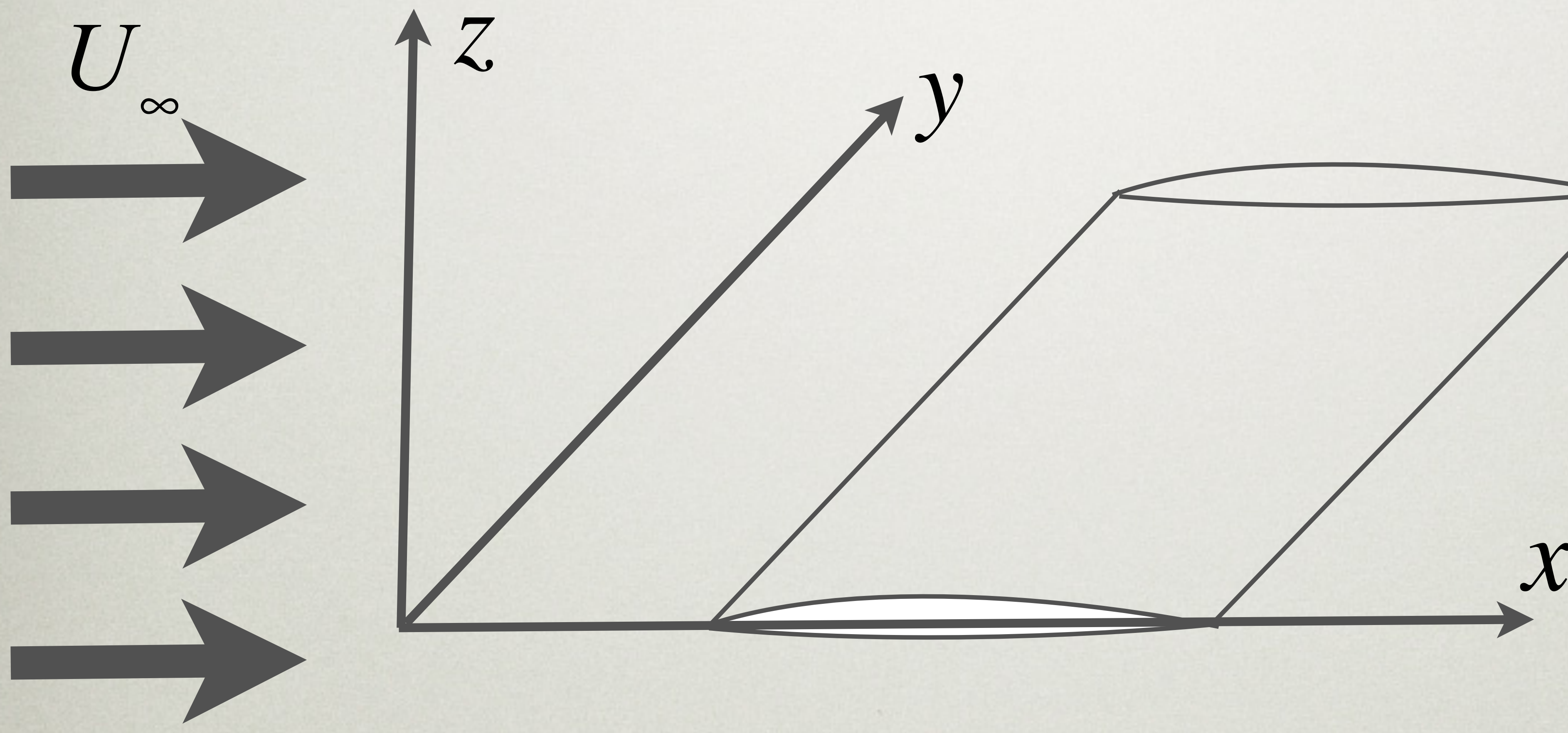
薄翼理論

Thin Wing Theory

適用対象



$$U_\infty \approx u >> v, w$$

$$a \approx a_\infty$$

速度ポテンシャル方程式

$$\left(1-\frac{u^2}{a^2}\right)\frac{\partial^2\Phi}{\partial x^2}+\left(1-\frac{v^2}{a^2}\right)\frac{\partial^2\Phi}{\partial y^2}+\left(1-\frac{w^2}{a^2}\right)\frac{\partial^2\Phi}{\partial z^2}$$

$$-\frac{2uv}{a^2}\frac{\partial^2\Phi}{\partial x\partial y}-\frac{2vw}{a^2}\frac{\partial^2\Phi}{\partial y\partial z}-\frac{2wu}{a^2}\frac{\partial^2\Phi}{\partial z\partial x}=0$$

$$U_\infty \approx u >> v, w$$

$$a \approx a_\infty$$

$$\approx\left(1-\frac{U_\infty^2}{a_\infty^2}\right)\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}$$

$$=\left(1-M_\infty^2\right)\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}$$

$$M_\infty=\frac{U_\infty}{a_\infty}$$

速度ポテンシャルの変換

$$\Phi = \phi + U_\infty x$$

一様流部分の分離

$$\left(1 - M_\infty^2\right)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

無限遠方での
境界条件

$$\phi \xrightarrow{x \to \pm\infty} 0$$

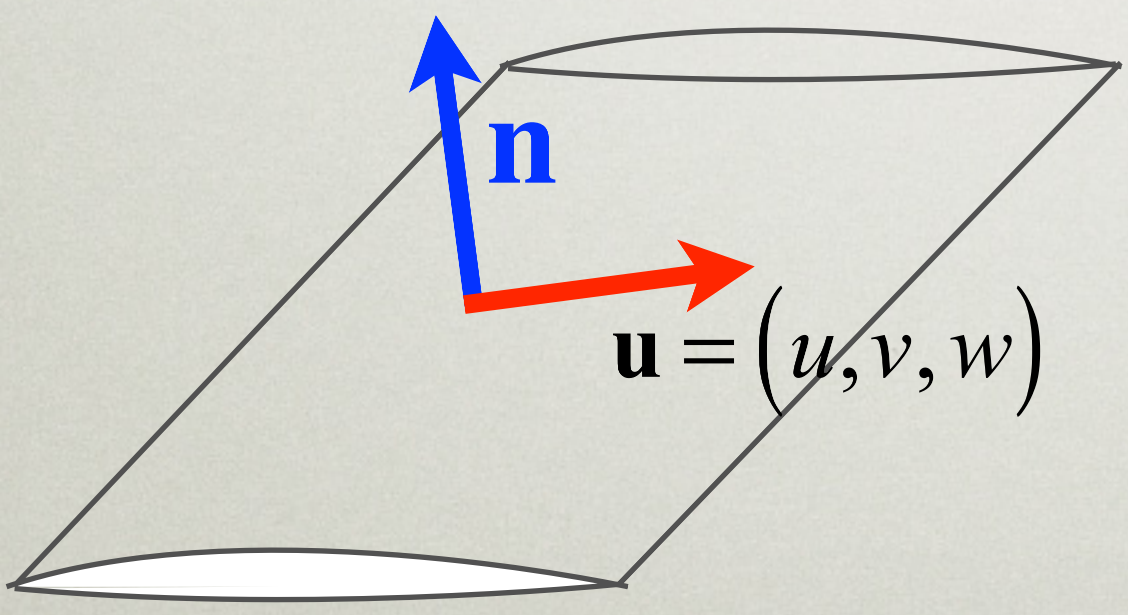
$$\xrightarrow{y \to \pm\infty} 0$$

$$\xrightarrow{z \to \pm\infty} 0$$

翼表面境界.1

翼面関数（既知）　$z-g(x,y)=0$

翼面法線ベクトル　$\mathbf{n}//\left(-\frac{dg}{dx},-\frac{dg}{dy},1\right)$



翼面法線ベクトルと
翼表面速度の関係

$$\mathbf{u}\perp\mathbf{n}$$

$$-u\frac{\partial g}{\partial x}-v\frac{\partial g}{\partial y}+w=0$$

翼表面境界.2

$$-u\frac{\partial g}{\partial x}-v\frac{\partial g}{\partial y}+w=0$$

$$u\approx U_\infty \qquad v<<u \qquad w=\frac{\partial \phi}{\partial z}$$

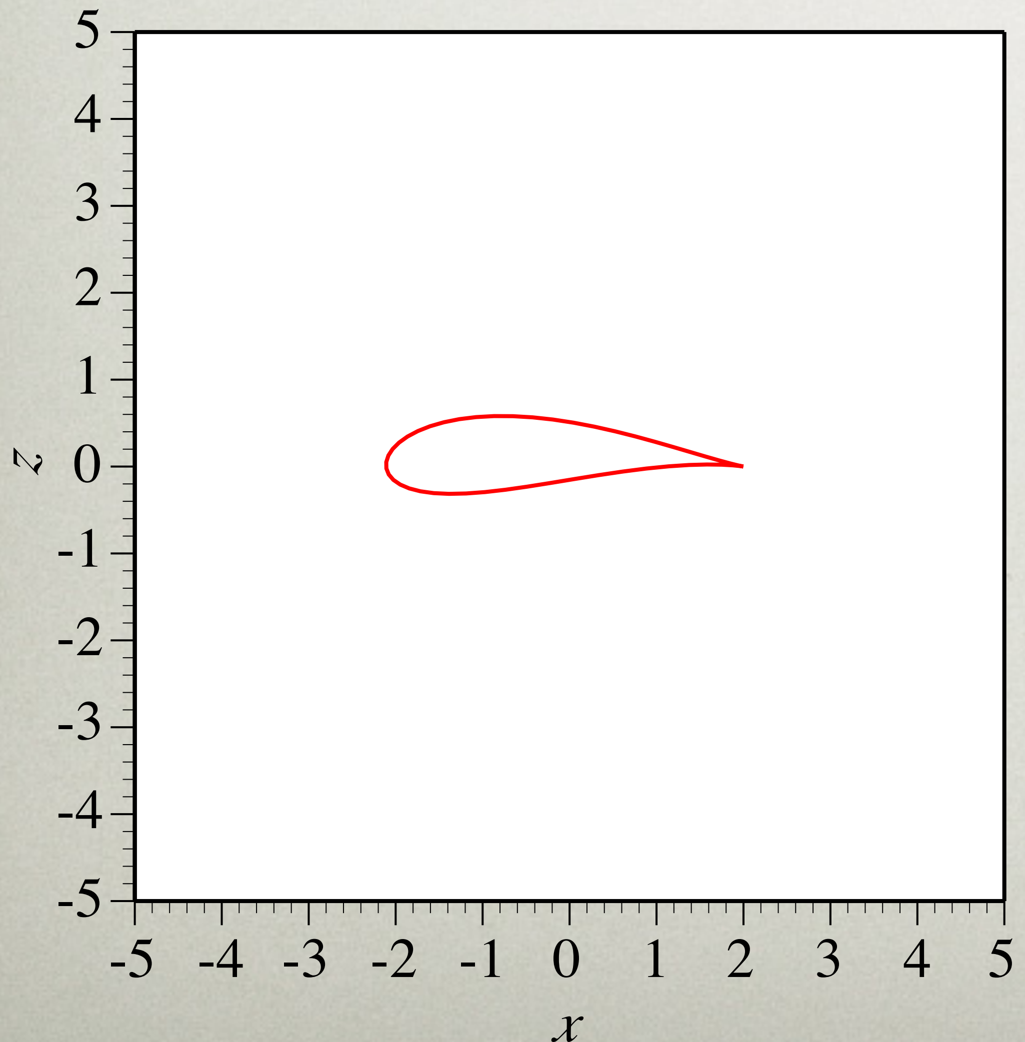
⇩

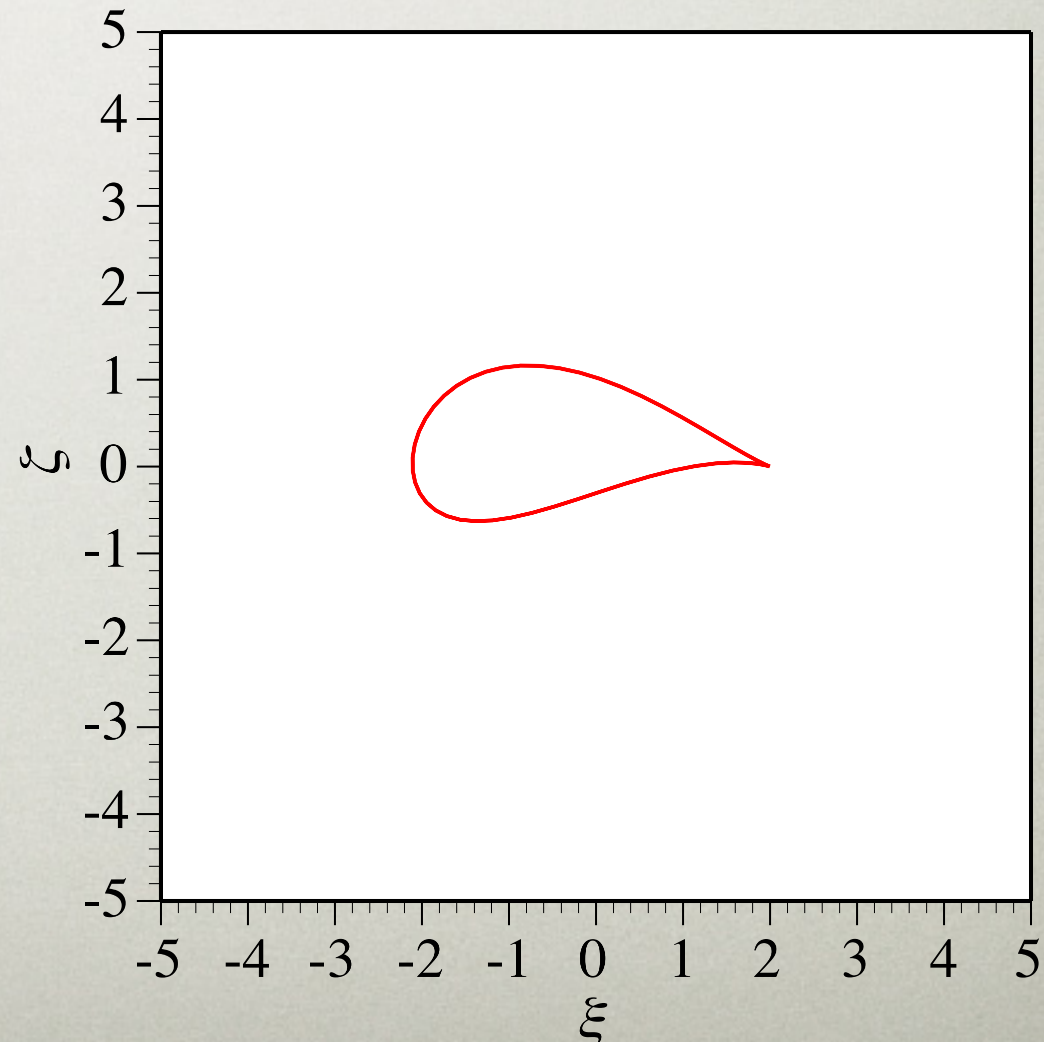
$$\frac{\partial \phi}{\partial z}=U_\infty\frac{\partial g}{\partial x}$$

座標変換

$$x = \xi,\ y = \mu\eta,\ z = \mu\zeta$$

$$\mu = \sqrt{1 - M_\infty^2}$$





微分作用素の変換

$$\frac{\partial}{\partial x}=\frac{d\xi}{dx}\frac{\partial}{\partial\xi}+\frac{d\eta}{dx}\frac{\partial}{\partial\eta}+\frac{d\zeta}{dx}\frac{\partial}{\partial\zeta}=\frac{dx}{dx}\frac{\partial}{\partial\xi}+\frac{d\mu y}{dx}\frac{\partial}{\partial\eta}+\frac{d\mu z}{dx}\frac{\partial}{\partial\zeta}=\frac{\partial}{\partial\xi}$$

$$\frac{\partial}{\partial y}=\frac{d\xi}{dy}\frac{\partial}{\partial\xi}+\frac{d\eta}{dy}\frac{\partial}{\partial\eta}+\frac{d\zeta}{dy}\frac{\partial}{\partial\zeta}=\frac{dx}{dy}\frac{\partial}{\partial\xi}+\frac{d\mu y}{dy}\frac{\partial}{\partial\eta}+\frac{d\mu z}{dy}\frac{\partial}{\partial\zeta}=\mu\frac{\partial}{\partial\eta}$$

$$\frac{\partial}{\partial z}=\frac{d\xi}{dz}\frac{\partial}{\partial\xi}+\frac{d\eta}{dz}\frac{\partial}{\partial\eta}+\frac{d\zeta}{dz}\frac{\partial}{\partial\zeta}=\frac{dx}{dz}\frac{\partial}{\partial\xi}+\frac{d\mu y}{dz}\frac{\partial}{\partial\eta}+\frac{d\mu z}{dz}\frac{\partial}{\partial\zeta}=\mu\frac{\partial}{\partial\zeta}$$

速度ポテンシャルの変換

$$\phi(x, y, z) = \lambda\phi_I(\xi, \eta, \zeta)$$

$$\frac{\partial^2\phi_I}{\partial\xi^2} + \frac{\partial^2\phi_I}{\partial\eta^2} + \frac{\partial^2\phi_I}{\partial\zeta^2} = 0$$

ラプラス方程式

⇒ 非圧縮性流れの解を使える

変換後の翼表面境界

$$\frac{\partial \phi_I}{\partial \zeta} = \frac{U_\infty}{\lambda\mu}\frac{\partial g(\xi, \mu\eta)}{\partial \xi} \Leftrightarrow \frac{\partial \phi}{\partial z} = U_\infty \frac{\partial g}{\partial x}$$

変換後の翼表面

$$\zeta = \frac{1}{\lambda\mu} g(\xi, \mu\eta)$$

翼に働く圧力の導出

ベリヌーイの定理

$$\frac{u^2 + \cancel{v^2 + w^2}}{2} + \frac{a^2}{\gamma - 1} = \frac{U_\infty^2}{2} + \frac{a_\infty^2}{\gamma - 1}$$

状態方程式

$$p = \rho^\gamma \exp\left(\frac{S - S_0}{C_V}\right)$$

音速定義式

$$a^2 = \gamma \frac{p}{\rho}$$

$$u = U_\infty + \frac{\partial \phi}{\partial x}$$

$$\frac{p - p_\infty}{\rho_\infty U_\infty^2 / 2} = -\frac{2}{U_\infty}\frac{\partial \phi}{\partial x} = -\frac{2\lambda}{U_\infty}\frac{\partial \phi_I}{\partial \xi}$$

Prandtl-Glauertの法則

- M_∞の圧縮性一様流れ中に置かれた薄い翼に働く圧力は、非圧縮性一様流れ中に翼幅をμ倍し、厚さを$1/\lambda\mu$倍した翼に働く圧力に等しい。