# $M^2$展開法の例

# 分かり易いがいい加減な$M^2$展開法

# 速度ポテンシャル方程式

定常流

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=\frac{u^2}{a^2}\frac{\partial^2\Phi}{\partial x^2}+\frac{v^2}{a^2}\frac{\partial^2\Phi}{\partial y^2}+\frac{w^2}{a^2}\frac{\partial^2\Phi}{\partial z^2}$$

$$+\frac{2uv}{a^2}\frac{\partial^2\Phi}{\partial x\partial y}+\frac{2vw}{a^2}\frac{\partial^2\Phi}{\partial y\partial z}+\frac{2wu}{a^2}\frac{\partial^2\Phi}{\partial z\partial x}$$

定常２次元流

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=\frac{u^2}{a^2}\frac{\partial^2\Phi}{\partial x^2}+\frac{v^2}{a^2}\frac{\partial^2\Phi}{\partial y^2}+\frac{2uv}{a^2}\frac{\partial^2\Phi}{\partial x\partial y}$$

# 円筒座標系での方程式

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2}=\frac{1}{a^2}\left\{\left(\frac{\partial \Phi}{\partial r}\right)^2\frac{\partial^2 \Phi}{\partial r^2}+\frac{1}{r^4}\left(\frac{\partial \Phi}{\partial \theta}\right)^2\frac{\partial^2 \Phi}{\partial \theta^2}\right.$$

$$\left.+\frac{2}{r^2}\frac{\partial \Phi}{\partial r}\frac{\partial \Phi}{\partial \theta}\frac{\partial^2 \Phi}{\partial r \partial \theta}-\frac{1}{r^3}\frac{\partial \Phi}{\partial r}\left(\frac{\partial \Phi}{\partial \theta}\right)^2\right\}$$

吹き出し吸い込み流れ

２重 吹き出し吸い込み流れ

# １次元ケース（動径方向）

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right)=\frac{1}{a^2}\left(\frac{\partial \Phi}{\partial r}\right)^2\frac{\partial^2 \Phi}{\partial r^2}$$

摂動展開

$$\Phi=\Phi_0+\delta\Phi_1+\delta^2\Phi_2+\cdots$$

摂動方程式

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right)=\delta\frac{1}{a^2}\left(\frac{\partial \Phi}{\partial r}\right)^2\frac{\partial^2 \Phi}{\partial r^2}$$

# １次元ケース（動径方向）

０次方程式 $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi_0}{\partial r}\right)=0$$

１次方程式 $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi_1}{\partial r}\right)=\frac{1}{a^2}\left(\frac{\partial \Phi_0}{\partial r}\right)^2\frac{\partial^2 \Phi_0}{\partial r^2}$$

２次方程式 $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi_2}{\partial r}\right)=\frac{1}{a^2}\left(\frac{\partial \Phi_0}{\partial r}\right)^2\frac{\partial^2 \Phi_1}{\partial r^2}+\frac{2}{a^2}\frac{\partial \Phi_1}{\partial r}\frac{\partial \Phi_0}{\partial r}\frac{\partial^2 \Phi_0}{\partial r^2}$$

# 吹出し吸込み解

0次解 $\Phi_0 = m\log r$

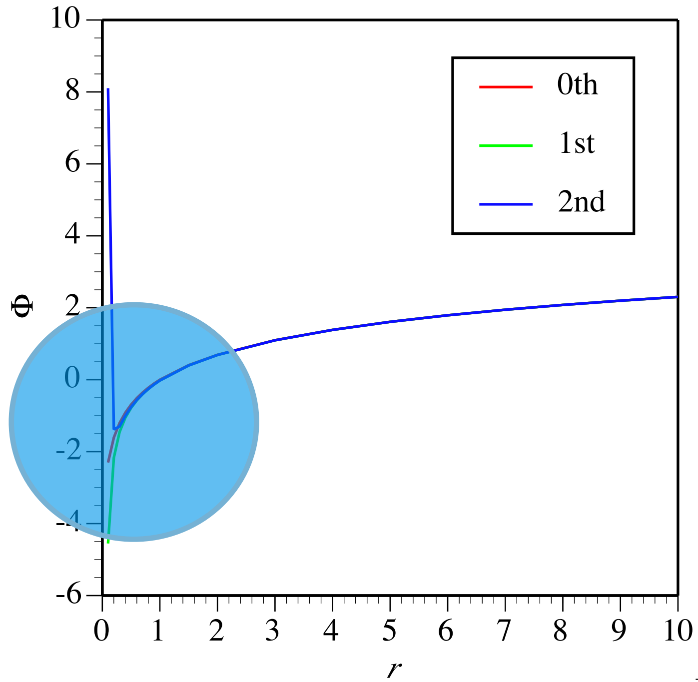
1次解 $\Phi_1 = -\frac{m^3}{4a^2}\frac{1}{r^2}$

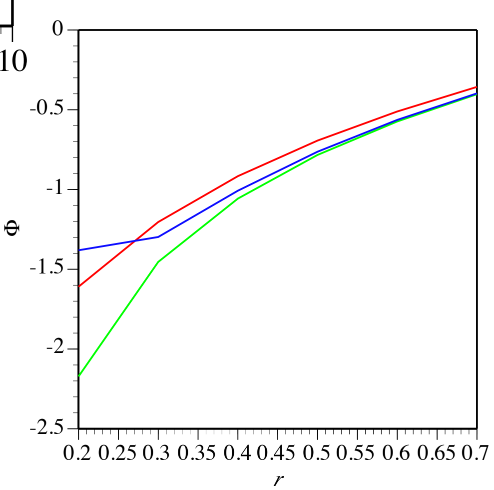
2次解 $\Phi_2 = \frac{5m^5}{32a^4}\frac{1}{r^4}$

解表現 $\Phi = m\left\{\log r - \frac{m^2}{4a^2}\frac{1}{r^2} + \frac{5m^4}{32a^4}\frac{1}{r^4} + \cdots\right\}$

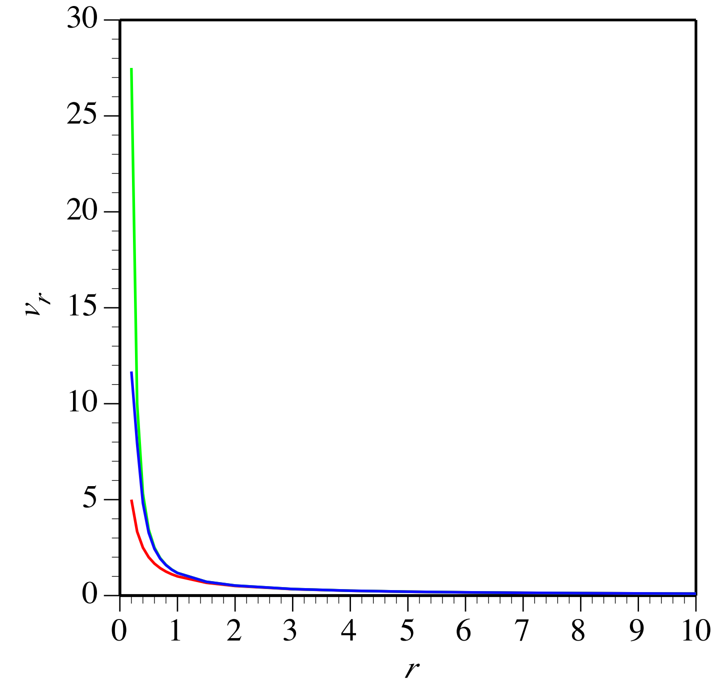
# 結果例

## 速度ポテンシャル




## 動径方向速度




$$v_r = \frac{\partial \Phi}{\partial r} = m\left\{\frac{1}{r} + \frac{m^2}{2a^2}\frac{1}{r^3} - \frac{5m^4}{8a^4}\frac{1}{r^5} + \cdots\right\}$$

# 2次元

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}=\frac{1}{a^2}\left\{\left(\frac{\partial\Phi}{\partial r}\right)^2\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^4}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\frac{\partial^2\Phi}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial\Phi}{\partial r}\frac{\partial\Phi}{\partial\theta}\frac{\partial^2\Phi}{\partial r\partial\theta}-\frac{1}{r^3}\frac{\partial\Phi}{\partial r}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\right\}$$

## 摂動展開

$$\Phi=\Phi_0+\delta\Phi_1+\delta^2\Phi_2+\cdots$$

## 摂動方程式

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}=\frac{\delta}{a^2}\left\{\left(\frac{\partial\Phi}{\partial r}\right)^2\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^4}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\frac{\partial^2\Phi}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial\Phi}{\partial r}\frac{\partial\Phi}{\partial\theta}\frac{\partial^2\Phi}{\partial r\partial\theta}-\frac{1}{r^3}\frac{\partial\Phi}{\partial r}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\right\}$$

# 2次元

0次方程式 $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi_0}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \Phi_0}{\partial \theta^2}=0$$

1次方程式

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi_1}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \Phi_1}{\partial \theta^2}=\frac{1}{a^2}\left\{\left(\frac{\partial \Phi_0}{\partial r}\right)^2\frac{\partial^2 \Phi_0}{\partial r^2}+\frac{1}{r^4}\left(\frac{\partial \Phi_0}{\partial \theta}\right)^2\frac{\partial^2 \Phi_0}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial \Phi_0}{\partial r}\frac{\partial \Phi_0}{\partial \theta}\frac{\partial^2 \Phi_0}{\partial r\partial \theta}-\frac{1}{r^3}\frac{\partial \Phi_0}{\partial r}\left(\frac{\partial \Phi_0}{\partial \theta}\right)^2\right\}$$

2次方程式

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi_2}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \Phi_2}{\partial \theta^2}=\frac{1}{a^2}\left\{\left(\frac{\partial \Phi_0}{\partial r}\right)^2\frac{\partial^2 \Phi_1}{\partial r^2}+2\frac{\partial \Phi_1}{\partial r}\frac{\partial \Phi_0}{\partial r}\frac{\partial^2 \Phi_0}{\partial r^2}+\frac{1}{r^4}\left(\frac{\partial \Phi_0}{\partial \theta}\right)^2\frac{\partial^2 \Phi_1}{\partial \theta^2}\right.$$

$$+\frac{2}{r^4}\frac{\partial \Phi_1}{\partial \theta}\frac{\partial \Phi_0}{\partial \theta}\frac{\partial^2 \Phi_0}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial \Phi_1}{\partial r}\frac{\partial \Phi_0}{\partial \theta}\frac{\partial^2 \Phi_0}{\partial r\partial \theta}+\frac{2}{r^2}\frac{\partial \Phi_0}{\partial r}\frac{\partial \Phi_1}{\partial \theta}\frac{\partial^2 \Phi_0}{\partial r\partial \theta}$$

$$\left.+\frac{2}{r^2}\frac{\partial \Phi_0}{\partial r}\frac{\partial \Phi_0}{\partial \theta}\frac{\partial^2 \Phi_1}{\partial r\partial \theta}-\frac{1}{r^3}\frac{\partial \Phi_1}{\partial r}\left(\frac{\partial \Phi_0}{\partial \theta}\right)^2-\frac{2}{r^3}\frac{\partial \Phi_0}{\partial r}\frac{\partial \Phi_1}{\partial \theta}\frac{\partial \Phi_0}{\partial \theta}\right\}$$

# 2次元

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}=\frac{1}{a^2}\left\{\left(\frac{\partial\Phi}{\partial r}\right)^2\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^4}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\frac{\partial^2\Phi}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial\Phi}{\partial r}\frac{\partial\Phi}{\partial\theta}\frac{\partial^2\Phi}{\partial r\partial\theta}-\frac{1}{r^3}\frac{\partial\Phi}{\partial r}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\right\}$$

## 摂動展開

$$\Phi=\Phi_0+\delta\Phi_1+\delta^2\Phi_2+\cdots$$

## 摂動方程式

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}=\frac{\delta}{a^2}\left\{\left(\frac{\partial\Phi}{\partial r}\right)^2\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^4}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\frac{\partial^2\Phi}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial\Phi}{\partial r}\frac{\partial\Phi}{\partial\theta}\frac{\partial^2\Phi}{\partial r\partial\theta}-\frac{1}{r^3}\frac{\partial\Phi}{\partial r}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\right\}$$

# 2次元

0次解 $$\Phi_0 = -\mu \frac{\cos\theta}{r}$$

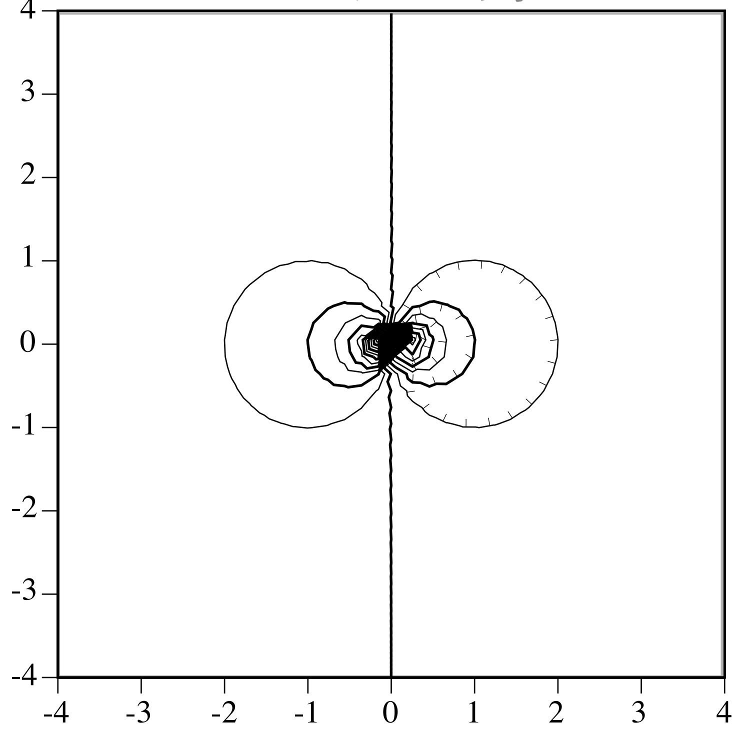
1次解 $$\Phi_1 = -\frac{\mu^3}{12a^2}\frac{\cos\theta}{r^5}$$

2次解 $$\Phi_2 = -\frac{\mu^5}{40a^4}\frac{\cos\theta}{r^{11}} - \frac{\mu^5}{36a^4}\frac{\cos^3\theta}{r^{11}}$$

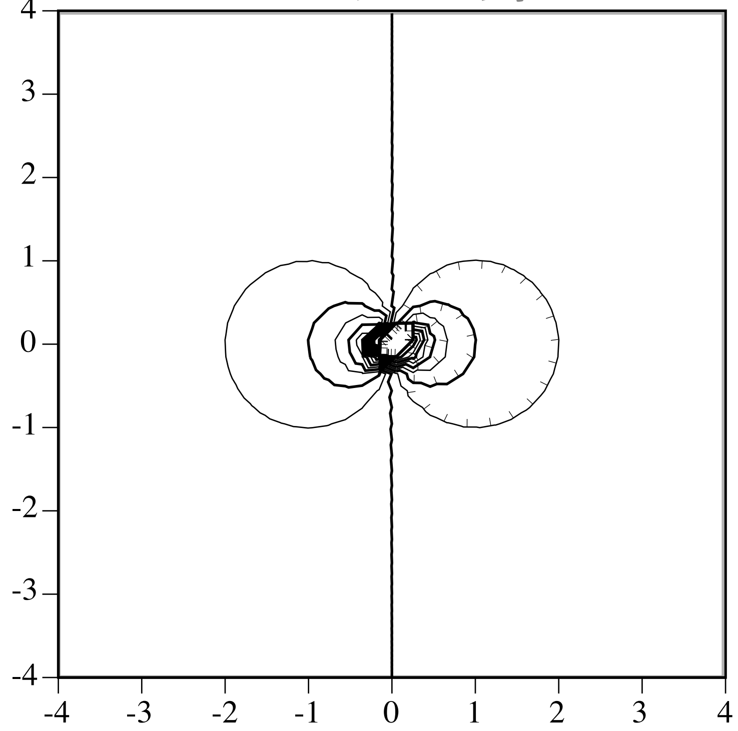
$$\Phi = -\mu\frac{\cos\theta}{r} - \frac{\mu^3}{12a^2}\frac{\cos\theta}{r^5} - \frac{\mu^5}{40a^4}\frac{\cos\theta}{r^{11}} - \frac{\mu^5}{36a^4}\frac{\cos^3\theta}{r^{11}} + \cdots$$

# ２重わき出し吸い込み流れ

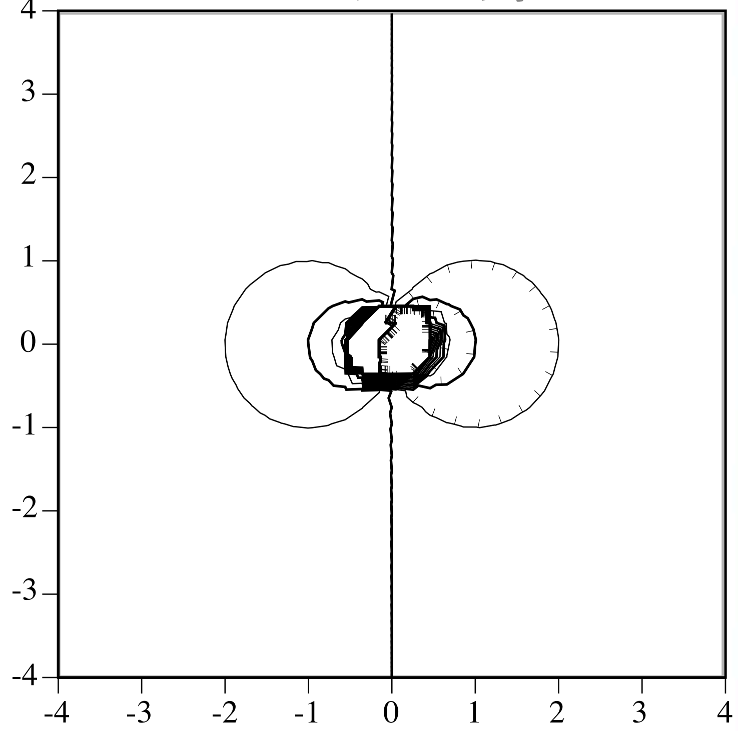
０次解




１次解




２次解



# 複素関数論を組み込んだ$M^2$展開法

# 複素速度ポテンシャル

$$W\left(z,z^*\right)=\Phi\left(z,z^*\right)+i\Psi\left(z,z^*\right)$$

$$\frac{\partial W\left(z,z^*\right)}{\partial z}=\frac{\partial \Phi\left(z,z^*\right)}{\partial z}+i\frac{\partial \Psi\left(z,z^*\right)}{\partial z}$$

$$\frac{\partial W\left(z,z^*\right)}{\partial z^*}=\frac{\partial \Phi\left(z,z^*\right)}{\partial z^*}+i\frac{\partial \Psi\left(z,z^*\right)}{\partial z^*}$$

複素共役を考慮

# 複素共役変換

$$\begin{pmatrix} z \\ z^* \end{pmatrix} = \begin{pmatrix} x+iy \\ x-iy \end{pmatrix} = \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{i}{2} \begin{pmatrix} -i & -i \\ -1 & 1 \end{pmatrix} \begin{pmatrix} z \\ z^* \end{pmatrix}$$

# 複素速度

$$\frac{\partial \Phi\left(z,z^*\right)}{\partial z}=\frac{1}{2}\frac{\partial \Phi}{\partial x}-\frac{i}{2}\frac{\partial \Phi}{\partial y}=\frac{1}{2}\left(u-iv\right)$$

$$i\frac{\partial \Psi\left(z,z^*\right)}{\partial z}=i\frac{1}{2}\frac{\partial \Psi}{\partial x}+\frac{1}{2}\frac{\partial \Psi}{\partial y}=\frac{1}{2}\frac{\rho}{\rho_\infty}\left(u-iv\right)$$

$$\frac{\partial W\left(z,z^*\right)}{\partial z}=\frac{1}{2}\left(1+\frac{\rho}{\rho_\infty}\right)\left(u-iv\right)=\frac{1}{2}\left(1+\frac{\rho}{\rho_\infty}\right)w$$

$$\frac{\partial W\left(z,z^*\right)}{\partial z^*}=\frac{1}{2}\left(1-\frac{\rho}{\rho_\infty}\right)\left(u+iv\right)=\frac{1}{2}\left(1-\frac{\rho}{\rho_\infty}\right)w^*$$

# 音速の空間依存性の導入

$$\frac{\partial \Phi(z,z^*)}{\partial z} = \frac{1}{2}(u - iv) \qquad \frac{\partial \Phi(z,z^*)}{\partial z^*} = \frac{1}{2}(u + iv)$$

$$\frac{\partial \Phi(z,z^*)}{\partial z}\frac{\partial \Phi(z,z^*)}{\partial z^*} = \frac{q^2}{4} \qquad q^2 = u^2 + v^2$$

音速式

$$a = a_\infty \left(\frac{\rho}{\rho_\infty}\right)^{\frac{\gamma-1}{2}}$$

ベルヌーイ式

$$\frac{1}{2}q^2 + \frac{a^2}{\gamma - 1} = \frac{1}{2}U_\infty^2 + \frac{a_\infty^2}{\gamma - 1}$$

# 音速と密度のマッハ数依存

$$\frac{a^2}{a_\infty^2} = 1 - \frac{\gamma - 1}{2}\frac{q^2}{a_\infty^2} + \frac{\gamma - 1}{2}\frac{U_\infty^2}{a_\infty^2}$$

音速

$$\frac{a}{a_\infty} = \sqrt{1 - \frac{\gamma - 1}{2} M_\infty^2 \left( \frac{q^2}{U_\infty^2} - 1 \right)}$$

密度

$$\frac{\rho}{\rho_\infty} = \left( 1 - \frac{\gamma - 1}{2} M_\infty^2 \left( \frac{q^2}{U_\infty^2} - 1 \right) \right)^{\frac{1}{\gamma - 1}}$$

# $M^2$展開法の導入

複素速度ポテンシャル

$$W = W_0 + M_\infty^2 W_1 + M_\infty^4 W_2 + \cdots$$

速度ポテンシャル

$$\Phi = \Phi_0 + M_\infty^2 \Phi_1 + M_\infty^4 \Phi_2 + \cdots$$

$$\frac{\partial \Phi(z,z^*)}{\partial z}\frac{\partial \Phi(z,z^*)}{\partial z^*} = \frac{q^2}{4}$$

$$= \left(\frac{\partial \Phi_0}{\partial z} + M_\infty^2 \frac{\partial \Phi_1}{\partial z} + M_\infty^4 \frac{\partial \Phi_2}{\partial z} + \cdots\right)\left(\frac{\partial \Phi_0}{\partial z^*} + M_\infty^2 \frac{\partial \Phi_1}{\partial z^*} + M_\infty^4 \frac{\partial \Phi_2}{\partial z^*} + \cdots\right)$$

$$= \frac{\partial \Phi_0}{\partial z}\frac{\partial \Phi_0}{\partial z^*} + M_\infty^2\left(\frac{\partial \Phi_1}{\partial z}\frac{\partial \Phi_0}{\partial z^*} + \frac{\partial \Phi_0}{\partial z}\frac{\partial \Phi_1}{\partial z^*}\right) + M_\infty^4\left(\frac{\partial \Phi_2}{\partial z}\frac{\partial \Phi_0}{\partial z^*} + \frac{\partial \Phi_1}{\partial z}\frac{\partial \Phi_1}{\partial z^*} + \frac{\partial \Phi_0}{\partial z}\frac{\partial \Phi_2}{\partial z^*}\right) + \cdots$$

# M²展開法の導入

$$q^2 = q_0^2 + M_\infty^2 q_1^2 + M_\infty^4 q_2^2 + \cdots$$

$$q_0^2 = 4\frac{\partial\Phi_0}{\partial z}\frac{\partial\Phi_0}{\partial z^*}$$

$$q_1^2 = 4\left(\frac{\partial\Phi_1}{\partial z}\frac{\partial\Phi_0}{\partial z^*} + \frac{\partial\Phi_0}{\partial z}\frac{\partial\Phi_1}{\partial z^*}\right)$$

$$q_2^2 = 4\left(\frac{\partial\Phi_2}{\partial z}\frac{\partial\Phi_0}{\partial z^*} + \frac{\partial\Phi_1}{\partial z}\frac{\partial\Phi_1}{\partial z^*} + \frac{\partial\Phi_0}{\partial z}\frac{\partial\Phi_2}{\partial z^*}\right)$$

# $M^2$展開法の導入

級数展開 $(1+x)^a = 1 + ax + \frac{1}{2!}a(a-1)x^2 + \cdots$

$$\frac{\rho}{\rho_\infty} = \left(1 - \frac{\gamma-1}{2}M_\infty^2\left(\frac{q^2}{U_\infty^2}-1\right)\right)^{\frac{1}{\gamma-1}}$$

$$= 1 - \frac{1}{2}M_\infty^2\left(\frac{q^2}{U_\infty^2}-1\right) + \frac{1}{2}\frac{1}{\gamma-1}\left(\frac{1}{\gamma-1}-1\right)\left(\frac{\gamma-1}{2}\right)^2 M_\infty^4\left(\frac{q^2}{U_\infty^2}-1\right)^2 + \cdots$$

$$= 1 - \frac{1}{2}M_\infty^2\left(\frac{q_0^2 + M_\infty^2 q_1^2}{U_\infty^2}-1\right) + \frac{1}{2}\frac{1}{\gamma-1}\left(\frac{1}{\gamma-1}-1\right)\left(\frac{\gamma-1}{2}\right)^2 M_\infty^4\left(\frac{q_0^2}{U_\infty^2}-1\right)^2 + \cdots$$

$$= 1 - \frac{1}{2}\left(\frac{q_0^2}{U_\infty^2}-1\right)M_\infty^2 - \left\{\frac{1}{2}\frac{q_1^2}{U_\infty^2} - \frac{(2-\gamma)}{8}\left(\frac{q_0^2}{U_\infty^2}-1\right)^2\right\}M_\infty^4 + \cdots = 1 - R_1 M_\infty^2 - R_2 M_\infty^4 + \cdots$$

# $M^2$展開法の導入

$$R_1 = \frac{1}{2}\left(\frac{q_0^2}{U_\infty^2} - 1\right)$$

$$R_2 = \frac{1}{2}\frac{q_1^2}{U_\infty^2} - \frac{(2-\gamma)}{8}\left(\frac{q_0^2}{U_\infty^2} - 1\right)^2$$

# M2展開法の導入

$$\frac{\partial W}{\partial z^*} = \left(1 - \frac{\rho}{\rho_\infty}\right)\frac{\partial \Phi}{\partial z^*} \qquad W = W_0 + M_\infty^2 W_1 + M_\infty^4 W_2 + \cdots$$

$$\frac{\partial W_0}{\partial z^*} + M_\infty^2 \frac{\partial W_1}{\partial z^*} + M_\infty^4 \frac{\partial W_2}{\partial z^*} + \cdots$$

$$= \left(R_1 M_\infty^2 + R_2 M_\infty^4 + \cdots\right)\left(\frac{\partial \Phi_0}{\partial z^*} + M_\infty^2 \frac{\partial \Phi_1}{\partial z^*} + M_\infty^4 \frac{\partial \Phi_2}{\partial z^*} + \cdots\right)$$

$$= M_\infty^2 \left(R_1 \frac{\partial \Phi_0}{\partial z^*} + R_2 M_\infty^2 \frac{\partial \Phi_0}{\partial z^*} + R_1 M_\infty^2 \frac{\partial \Phi_1}{\partial z^*} + \cdots\right)$$

$$= 0 + M_\infty^2 R_1 \frac{\partial \Phi_0}{\partial z^*} + M_\infty^4 \left(R_2 \frac{\partial \Phi_0}{\partial z^*} + R_1 \frac{\partial \Phi_1}{\partial z^*}\right) + \cdots$$

# 摂動方程式

$$\frac{\partial W_0}{\partial z^*} = 0$$

$$\frac{\partial W_1}{\partial z^*} = R_1 \frac{\partial \Phi_0}{\partial z^*}$$

$$\frac{\partial W_2}{\partial z^*} = R_2 \frac{\partial \Phi_0}{\partial z^*} + R_1 \frac{\partial \Phi_1}{\partial z^*}$$

# 摂動解

$$W_0 = f_0(z)$$

$$W_1 = \int dz^* R_1 \frac{\partial \Phi_0}{\partial z^*} + f_1(z)$$

$$W_2 = \int dz^* R_2 \frac{\partial \Phi_0}{\partial z^*} + \int dz^* R_1 \frac{\partial \Phi_1}{\partial z^*} + f_2(z)$$

$$R_1 = \frac{1}{2}\left(\frac{q_0^2}{U_\infty^2} - 1\right), R_2 = \frac{1}{2}\frac{q_1^2}{U_\infty^2} - \frac{(2-\gamma)}{8}\left(\frac{q_0^2}{U_\infty^2} - 1\right)^2$$

# 円柱周りの流れ

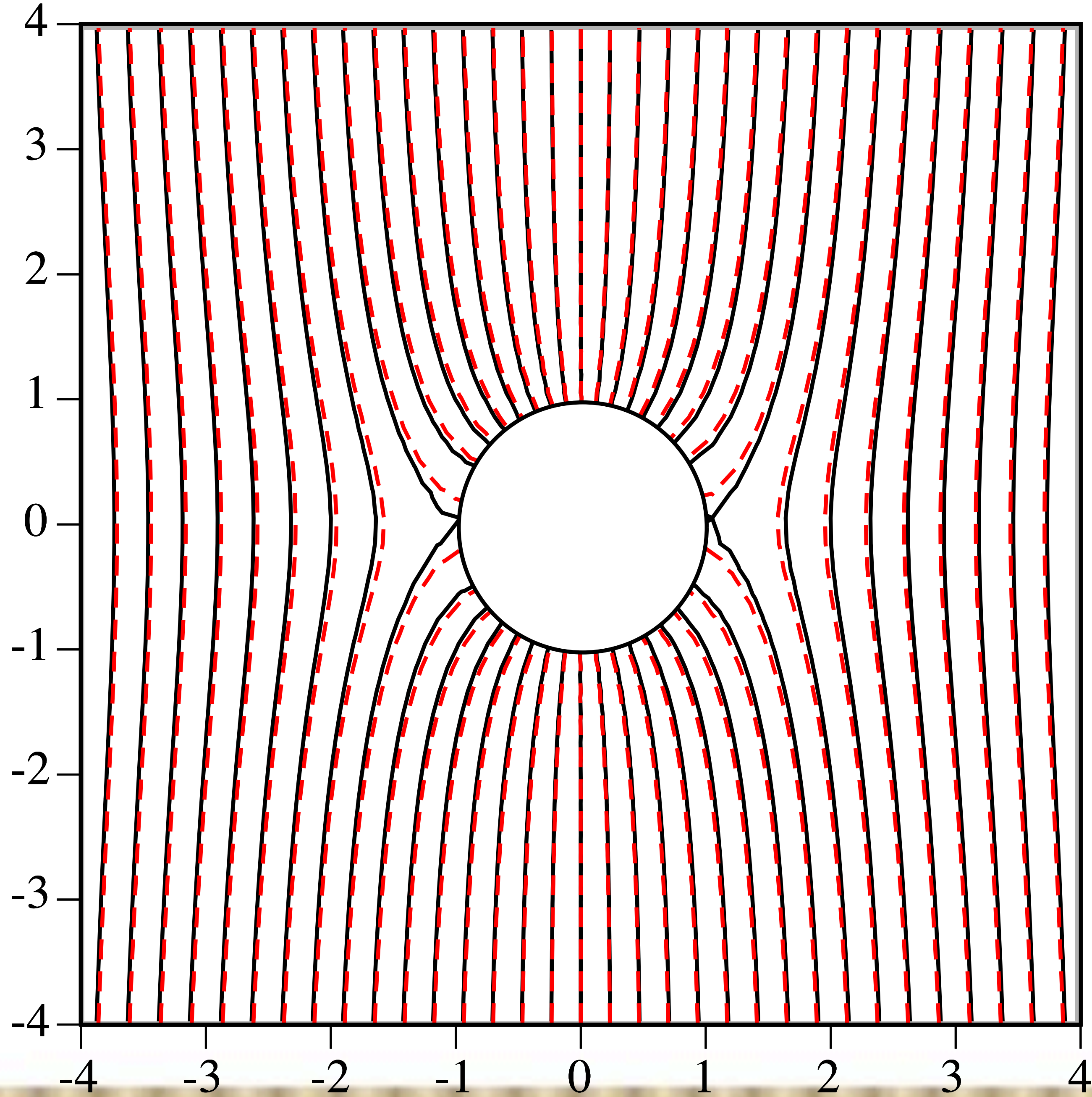
$$W_0 = f_0(z) = z + \frac{1}{z}$$

$$W_1 = \frac{1}{4}\left(\frac{1}{z^*} - \frac{1}{3z^{*3}} - \frac{z^*}{z^2} - \frac{2}{z^2 z^*} + \frac{1}{3z^2 z^{*3}}\right) + \frac{5}{6z} + \frac{1}{6z^3}$$

$$\Phi = \left(r + \frac{1}{r}\right)\cos\theta$$

$$+M_\infty^2\left\{\left(\frac{11}{6r} - \frac{3}{4r^3} + \frac{1}{12r^5}\right)\cos\theta - \left(\frac{1}{r} - \frac{1}{3r^3}\right)\cos^3\theta\right\} + \cdots$$

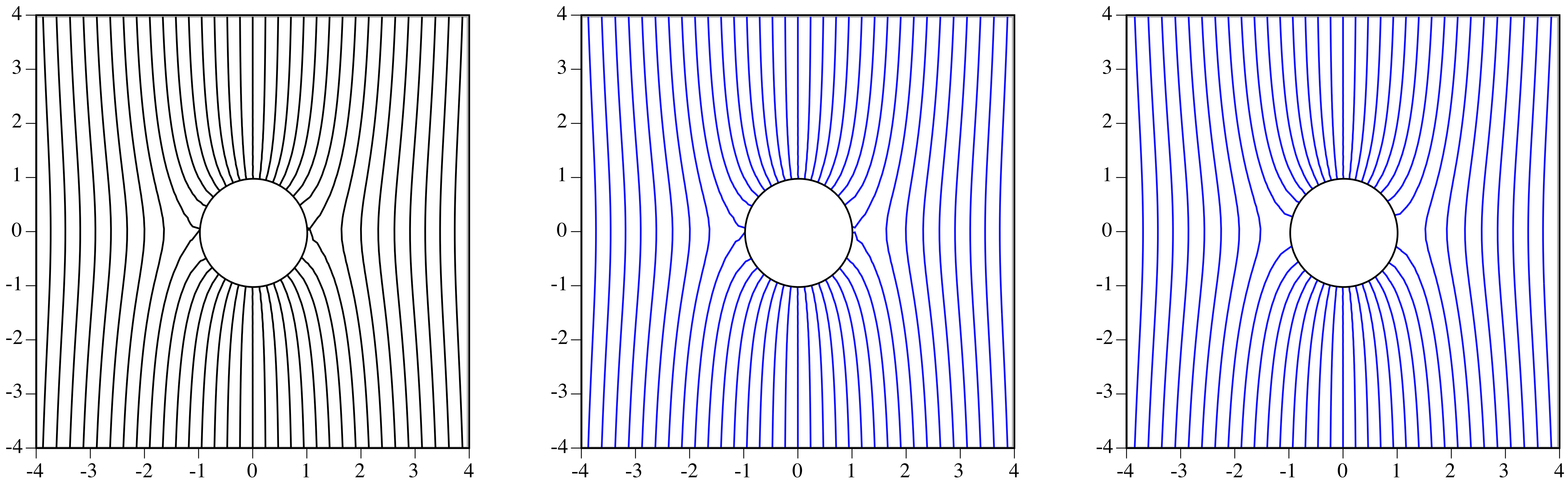
# 速度ポテンシャルの修正




M=0.3

# 速度ポテンシャル

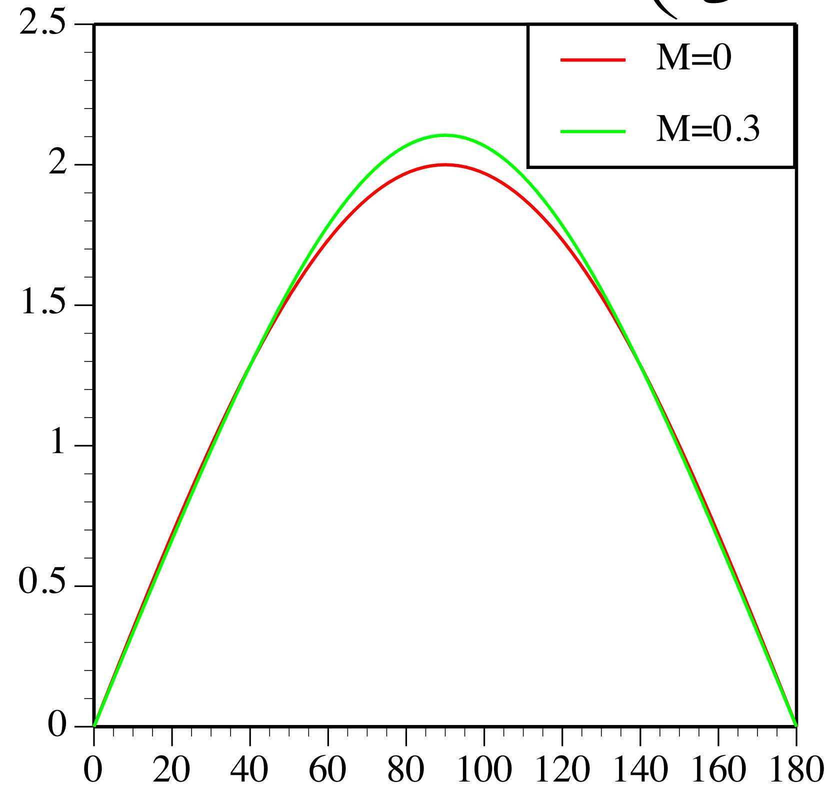
M=0　　M=0.1　　M=0.4



Mが大きくなると違いが明瞭に。

# 円柱表面速度分布

$$v_\theta(r=1,\theta)=2\sin\theta+M_\infty^2\left(\frac{2}{3}\sin\theta-\frac{1}{2}\sin 3\theta\right)$$




ややは円柱上および下部で速くなる。

# 流れ関数の摂動解

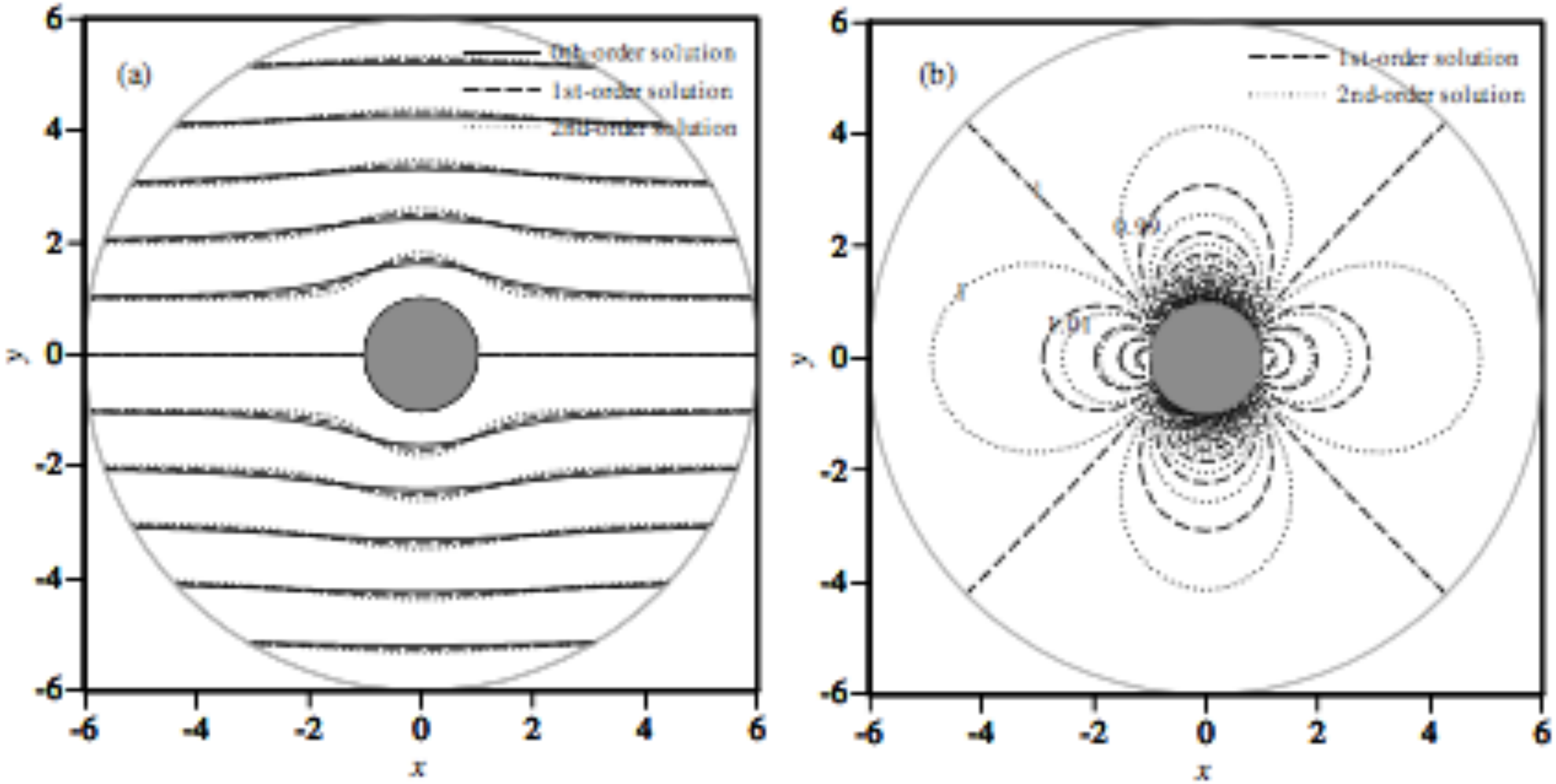
$$\Psi_0(r,\theta) = U_\infty r \sin\theta - \frac{U_\infty R^2}{r}\sin\theta \qquad (3.122)$$

$$\Psi_1(r,\theta) = U_\infty\left\{\left(-\frac{7}{12}\frac{R^2}{r} + \frac{1}{2}\frac{R^4}{r^3} + \frac{1}{12}\frac{R^6}{r^5}\right)\sin\theta + \left(\frac{1}{4}\frac{R^2}{r} - \frac{1}{4}\frac{R^4}{r^3}\right)\sin 3\theta\right\} \qquad (3.133)$$

$$\begin{aligned}\Psi_2(r,\theta) = U_\infty \Bigg[ &-\left\{\left(\frac{319}{240} - \frac{17h_1}{30}\right)\frac{R^2}{r} - \left(\frac{35}{24} - \frac{h_1}{4}\right)\frac{R^4}{r^3}\right. \\ &\left. + \left(\frac{1}{16} + \frac{h_1}{6}\right)\frac{R^6}{r^5} + \left(\frac{1}{12} + \frac{h_1}{8}\right)\frac{R^8}{r^7} - \left(\frac{1}{60} - \frac{h_1}{40}\right)\frac{R^{10}}{r^9}\right\}\sin\theta \\ &+\left\{\frac{13}{48}\frac{R^2}{r} - \left(\frac{2}{45} + \frac{17h_1}{40}\right)\frac{R^4}{r^3} - \left(\frac{1}{4} - \frac{3h_1}{8}\right)\frac{R^6}{r^5}\right. \\ &\left. + \left(\frac{1}{60} + \frac{h_1}{20}\right)\frac{R^8}{r^7} + \frac{1}{144}\frac{R^{10}}{r^9}\right\}\sin 3\theta \\ &-\left\{\frac{1}{16}\frac{R^2}{r} - \frac{h_1}{8}\frac{R^4}{r^3} - \left(\frac{1}{16} - \frac{h_1}{8}\right)\frac{R^6}{r^5}\right\}\sin 5\theta \Bigg] \end{aligned} \qquad (3.134)$$

$h_1$は比熱比の関数

# 結果（流線図）



(a)
0th-order solution
1st-order solution
2nd-order solution
6
4
2
0
-2
-4
-6
y
x
(b)
1st-order solution
2nd-order solution